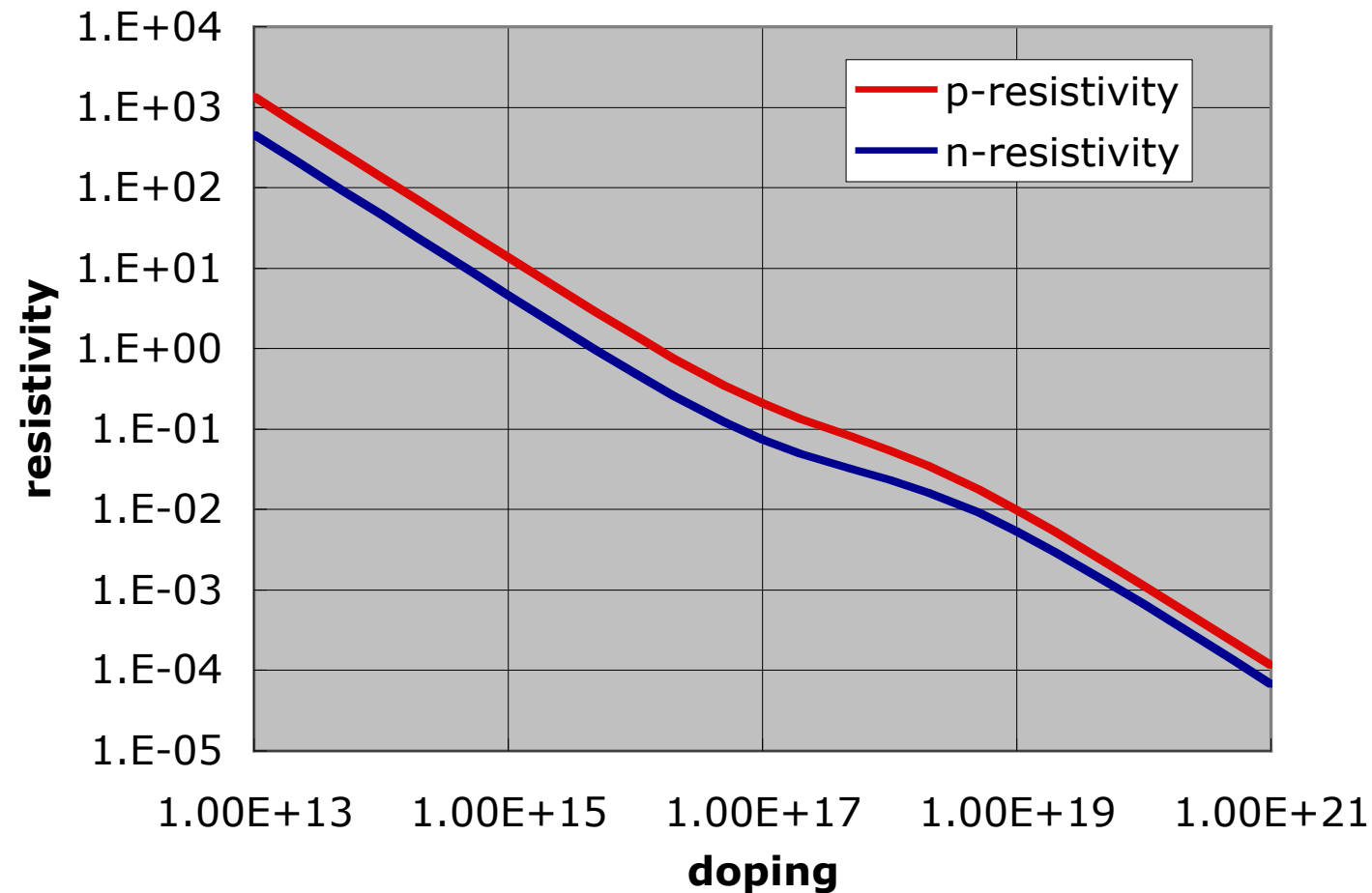
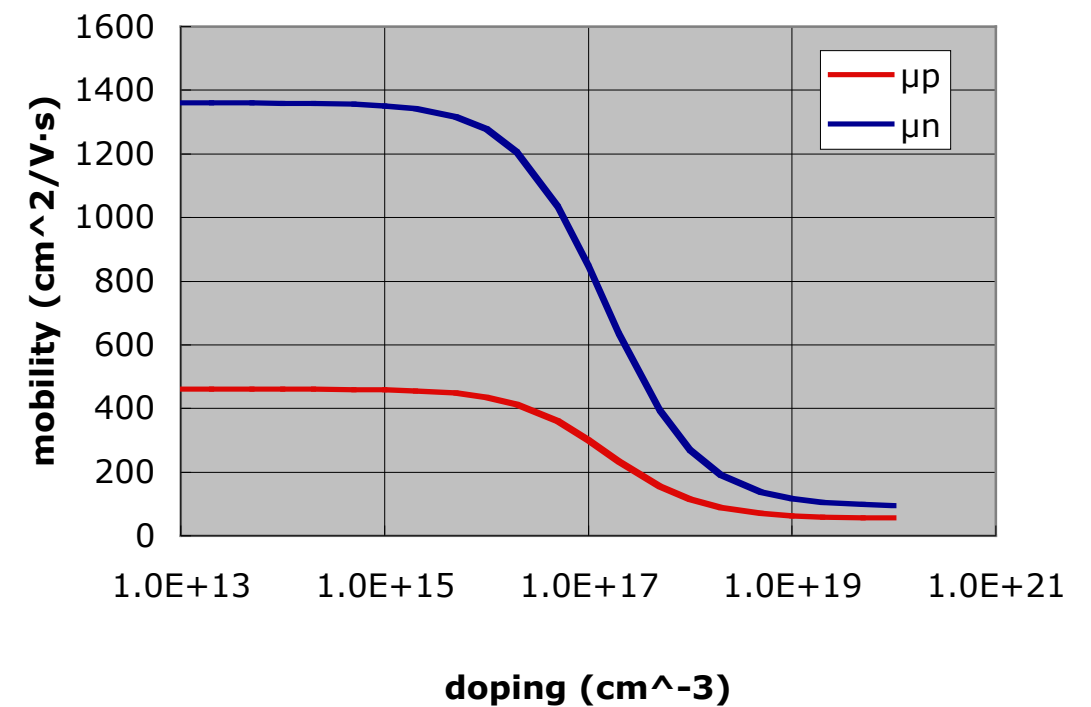


# Resistivity and resistance measurements

**silicon resistivity**



**silicon electron and hole mobility**

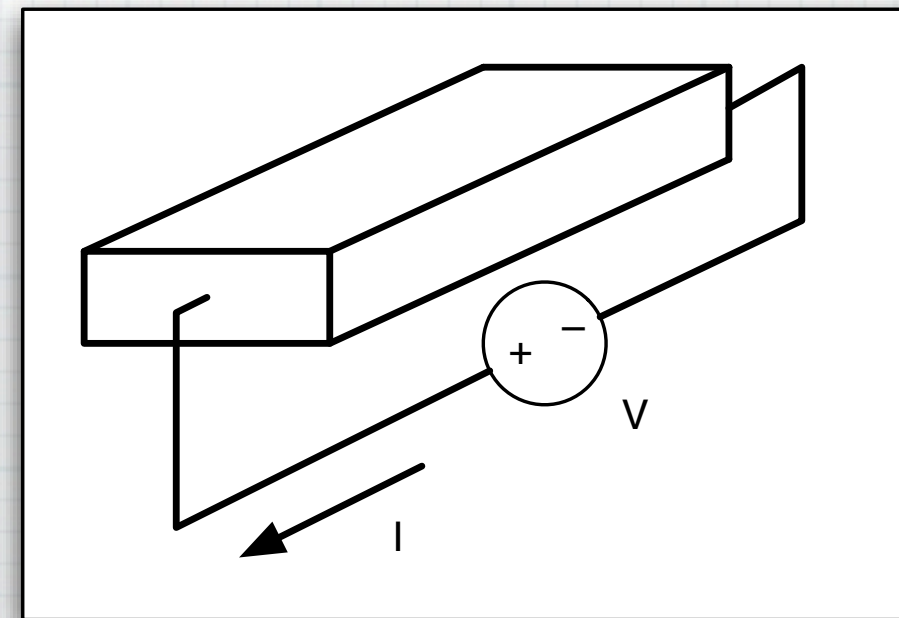
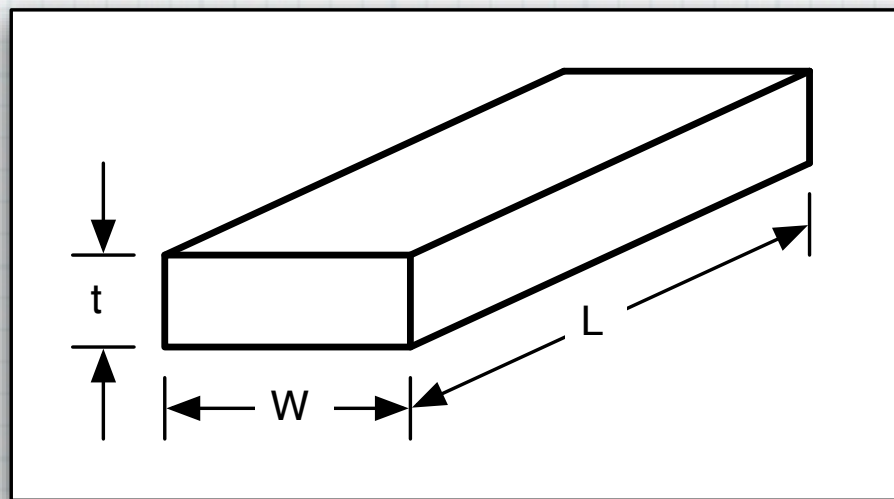


resistivity:  $\rho = \frac{1}{\sigma} = \frac{1}{q\mu N}$

Reference text: "Semiconductor material and device characterization 2/e" by Dieter Schroder, John Wiley, 1998.)

To measure the doping level, we can make use of simple resistance measurements.

EE 201 approach: Make a text-book resistor from a chunk of  $n$ -type material that has a uniform doping concentration,  $N_D$ . (The exact same approach applies to  $p$ -type material, as well.) The dimensions of the sample are  $W \times L \times t$ . The current will be carried by electrons, which have a concentration of  $n = N_D$ .



$$R = \frac{V}{I} = \frac{\rho}{t} \frac{L}{W} = \frac{\rho}{t} \cdot g_r$$

There may be problems with contact resistance.



# Four-point probe method

Put 4 probes in a collinear arrangement onto the sample surface. The probes are equally spaced. The wafer thickness is  $t$ .

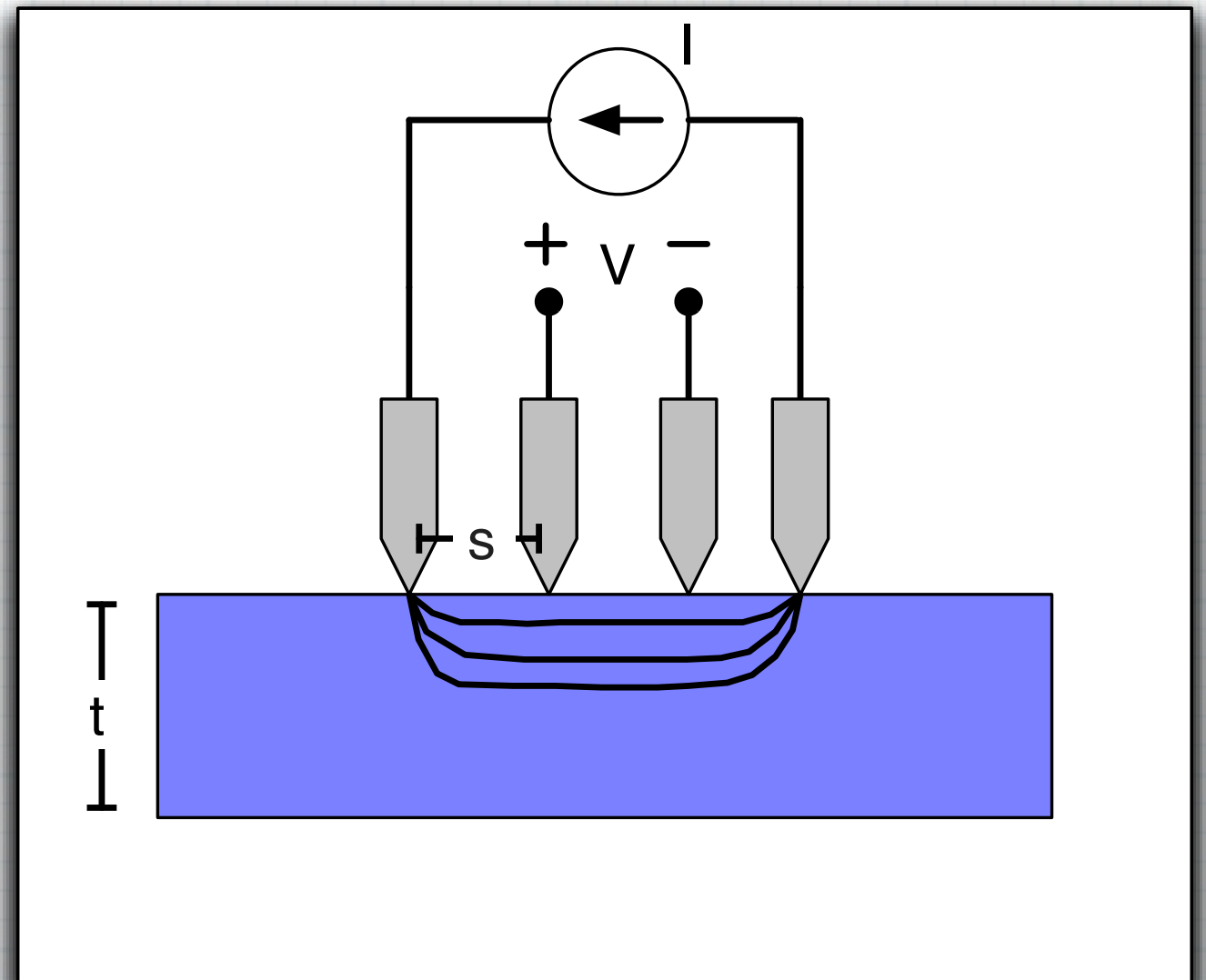
Pass a current between the outer two probes.

Measure the voltage between the inner two probes.

The use of separate current and voltage contacts gets away from problems with contact resistance.

The ratio of the voltage to the current will give a quantity like a resistance. Not surprisingly, the resistance is the product of the sample resistivity and the a geometrical factor.

$$R = \frac{V}{I} = \frac{\rho}{t} \cdot g_4$$



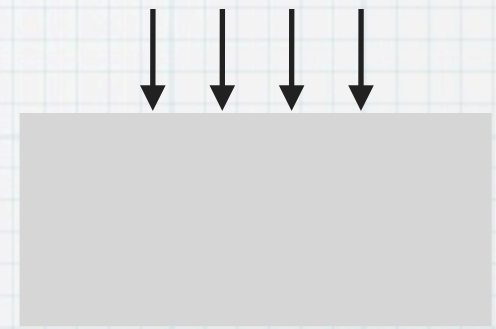


$$R = \frac{\rho}{t} \cdot g_4 \quad g_4 = \frac{1}{\pi} \ln \left[ \frac{\sinh(t/s)}{\sinh(t/2s)} \right] \quad \sinh(x) = \frac{1}{2} (e^x - e^{-x})$$

The geometric factor accounts for how much the current is “squished” in the layer as it flows between two outer probes. Depends on the ratio  $t/s$ .

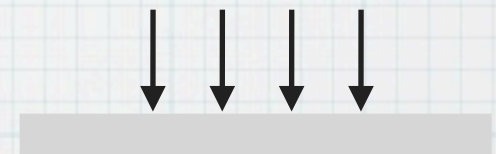
Thick wafer

$$\frac{t}{s} \gg 1 \quad g_4 \rightarrow \frac{t}{2\pi s} \quad R = \frac{\rho}{2\pi s}$$



Thin wafer (or diffused layer at the top surface of a wafer)

$$\frac{t}{s} \ll 1 \quad g_4 \rightarrow \frac{\ln(2)}{\pi} \quad R = \frac{\ln(2)}{\pi} \frac{\rho}{t}$$



Most starting wafers fall somewhere in between.

There are other corrections to account for edge effects, etc.

In lab, the probe spacing  $s = 40 \text{ mils} = 1.02 \text{ mm}$ .



# Example

In a four-point probe measurement on a silicon wafer that is uniformly doped n-type, the measured resistance is  $40\ \Omega$ . If wafer is  $400\ \mu\text{m}$  thick and the probe spacing is  $1\ \text{mm}$ , determine the wafer resistivity and doping concentration.

$$g_4 = \frac{1}{\pi} \ln \left[ \frac{\sinh(t/s)}{\sinh(t/2s)} \right] = \frac{1}{\pi} \ln \left[ \frac{\sinh(0.4\text{mm}/1\text{mm})}{\sinh(0.4\text{mm}/2\text{mm})} \right] = 0.2269$$

$$\rho = \frac{R \cdot t}{g_4} = \frac{(40\Omega)(0.04\text{cm})}{0.2269} = 7.05\Omega \cdot \text{cm}$$

From the graph:

$$N_D \approx 9 \times 10^{14} \text{cm}^{-3}$$

Also, have a look at: <http://solecon.com/sra/rho2ccal.htm>



# Sheet resistance

Look again at the resistance equation with uniform resistivity.

$$R = \frac{\rho}{t} \cdot g$$

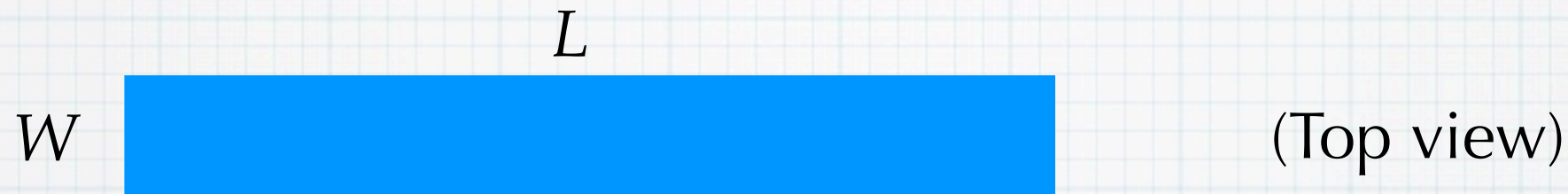
The quantity  $\rho/t$  has units of resistance. We note that it is a property of the particular layer (doping and thickness). This quantity is important enough that we give it a new name: the sheet resistance. The units typically given are  $\Omega/\square$  (ohms per square).

$$R_s = \frac{\rho}{t}$$

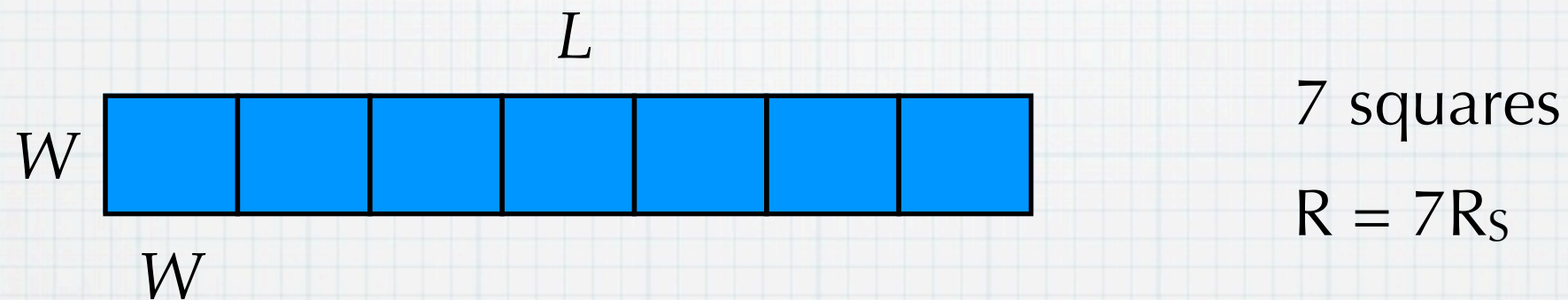
Using sheet resistance hides the fact that the resistor has thickness. If we know sheet resistance, then we need only to specify the resistor dimensions on the surface of the wafer, i.e. the dimensions determined by the mask.



Going back to the simple rectangular resistor:



$$R = \frac{\rho}{t} \frac{L}{W} = R_S \frac{L}{W}$$



Integrated circuit designers need to show the sheet resistances of the layers in order to make resistors. If  $R_S$  is known, then specifying  $L$  and  $W$  determines the resistance.



# Non-uniform doping profiles and sheet resistance

Things become a bit more complicated when the doping is not uniform. Since doping depends on depth, then resistivity depends on depth.

$$N(x) \rightarrow \rho(x) = \frac{1}{q\mu N(x)}$$

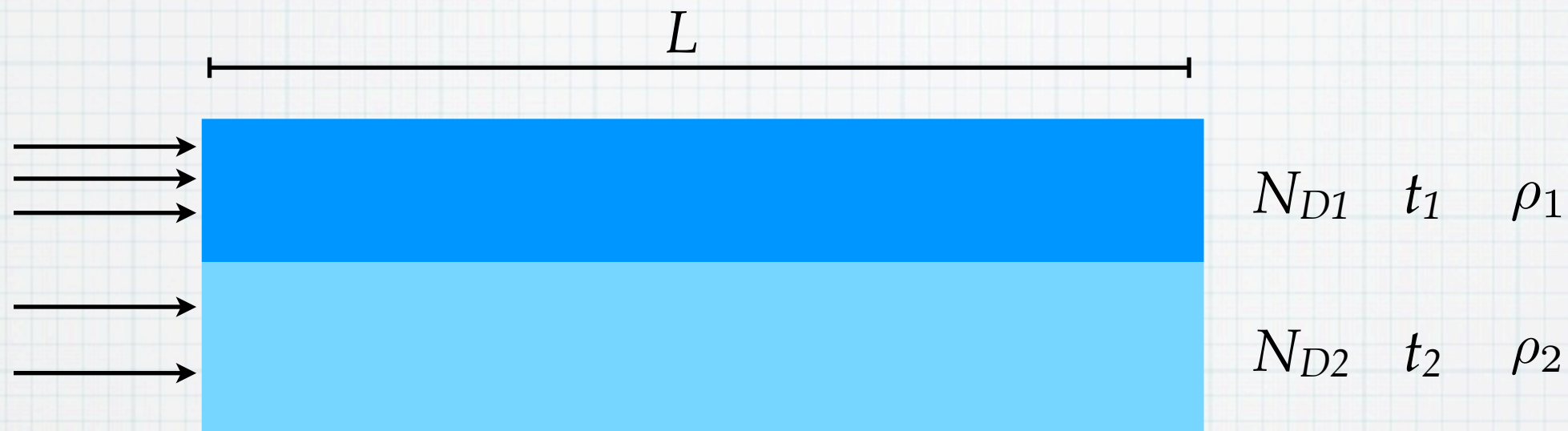
As we run a current through this non-uniform layer, the carrier spread themselves out according to the *local* resistivity, with the effects being averaged over the thickness of the layer.



More current flows in heavily doped regions; less current in lightly doped areas.



As a first step in seeing how to handle non-uniform doping, consider simple two-layer structure. Each layer is uniformly doped.



$$R_1 = \frac{\rho_1}{t_1} \frac{L}{W} \quad R_2 = \frac{\rho_2}{t_2} \frac{L}{W}$$

The two resistors are in parallel.

$$\begin{aligned} R_T^{-1} &= R_1^{-1} + R_2^{-1} \\ &= \frac{t_1}{\rho_1} \frac{W}{L} + \frac{t_2}{\rho_2} \frac{W}{L} = \left[ \frac{t_1}{\rho_1} + \frac{t_2}{\rho_2} \right] \frac{W}{L} = (R_S^{-1}) \frac{W}{L} \end{aligned}$$



$$R_S^{-1} = \frac{t_1}{\rho_1} + \frac{t_2}{\rho_2}$$

$$R_S^{-1} = q\mu_1 N_{D1}t_1 + q\mu_2 N_{D2}t_2$$

Of course, we can take this notion of two layers and extend it to any number of uniformly doped layers:

$$R_S^{-1} = q\mu_1 N_{D1}t_1 + q\mu_2 N_{D2}t_2 + q\mu_3 N_{D3}t_3 + q\mu_i N_{Di}t_i + \dots$$

$$R_S^{-1} = q \sum_{i=1} \mu_i N_{Di}t_i$$



We now have a path to finding the sheet resistance of a non-uniform doping profile. By conceptually breaking the profile into many small slices of thickness  $\Delta x$ , and treating each slice as having uniform doping,

$$R_S^{-1} = q \sum_{i=1}^{x_i < x_j} \mu(x_i) N(x_i) \Delta x$$

Of course, this is just a discrete representation of a “sheet-resistance integral”:

$$R_S^{-1} = q \int_0^{x_j} \mu(x) N(x) dx$$

This is not an easy computation because of the mobility variation. Generally, it requires numerical integration.



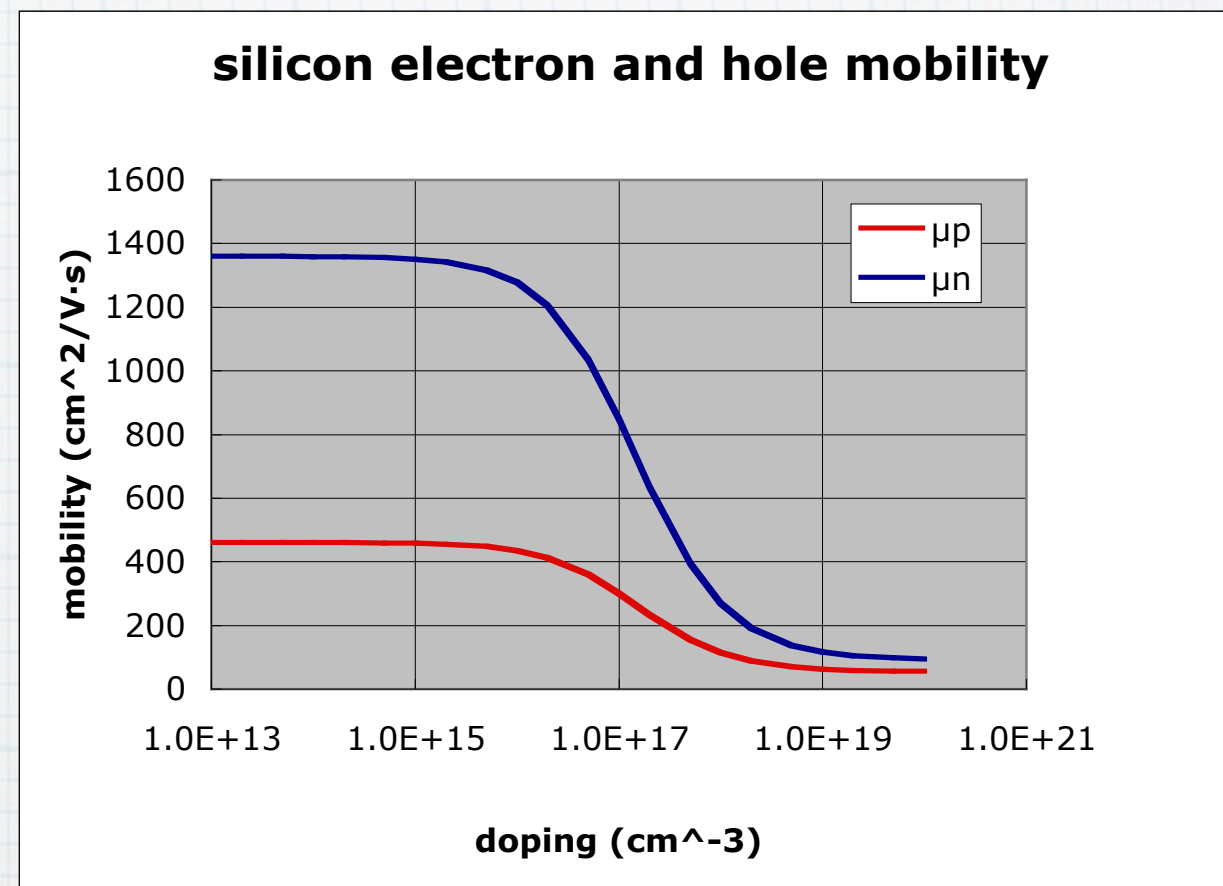
If mobility were constant, then the calculation would be straightforward (even familiar)

$$R_S^{-1} = q\bar{\mu} \int_0^{x_j} N(x) dx \quad \bar{\mu} \text{ is an "effective mobility"}$$

$$= q\bar{\mu}Q \quad \text{where } Q \text{ is the dose of the doped layer.}$$

But mobility is not a constant.

The mobility variation must be included in the integral, and this is messy.



$$\mu_p = 54.3 \frac{cm^2}{V \cdot s} + \left[ 406.9 \frac{cm^2}{V \cdot s} \right] \left( 1 + \frac{N_A}{2.35 \times 10^{17} cm^{-3}} \right)^{-0.88}$$

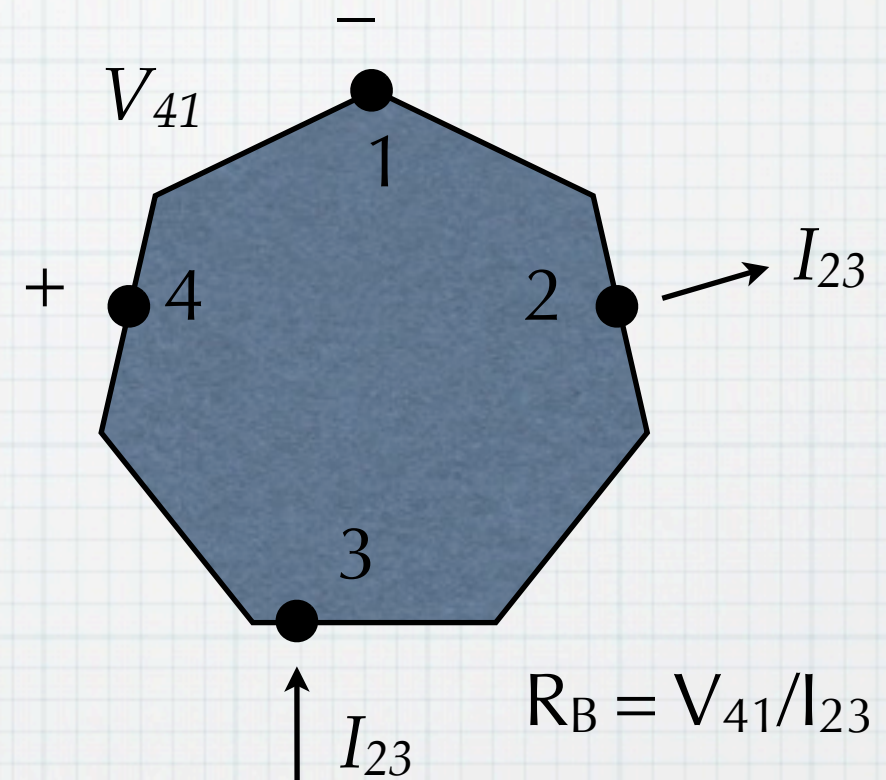
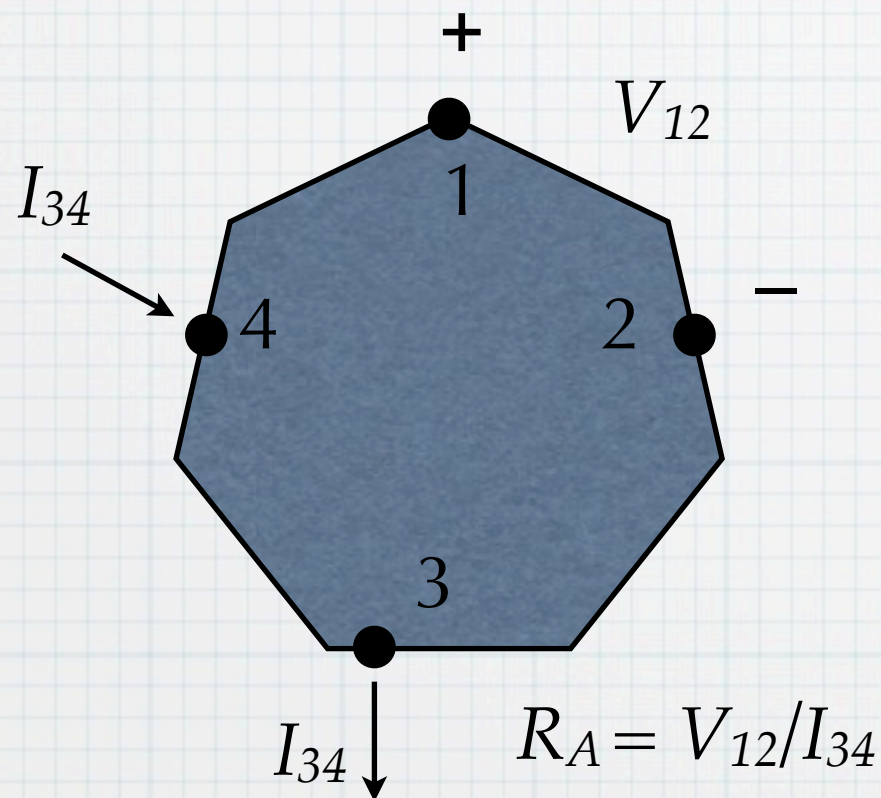
$$\mu_n = 92 \frac{cm^2}{V \cdot s} + \left[ 1268 \frac{cm^2}{V \cdot s} \right] \left( 1 + \frac{N_D}{1.3 \times 10^{17} cm^{-3}} \right)^{-0.91}$$



# van der Pauw measurements

Mr. van der Pauw showed that you don't need to know the shape of a sample to determine the sheet resistance, etc. He proved that a thin-layer sample could have any shape, if you make the four contacts very small and keep them at the very perimeter.

Then you perform 2 resistance-like measurements:



Sheet resistance is related to  $R_A$  and  $R_B$  through the formula:

$$\exp \left[ \frac{-\pi R_A}{R_S} \right] + \exp \left[ \frac{-\pi R_B}{R_S} \right] = 1$$



$$\exp \left[ \frac{-\pi R_A}{R_S} \right] + \exp \left[ \frac{-\pi R_B}{R_S} \right] = 1$$

If the sample and the placement of the contacts were both perfectly symmetric, then we expect  $R_A = R_B$  and we can compute sheet resistance easily:

$$R_S = \frac{\pi}{\ln(2)} R_A$$

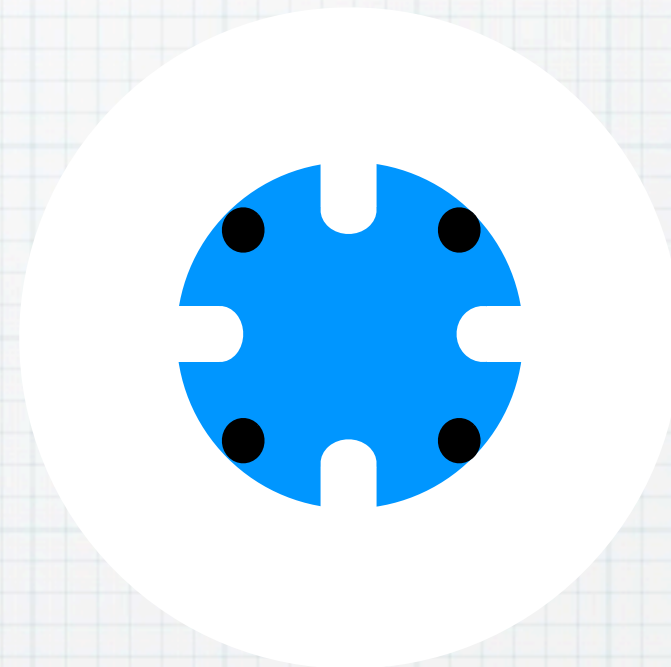
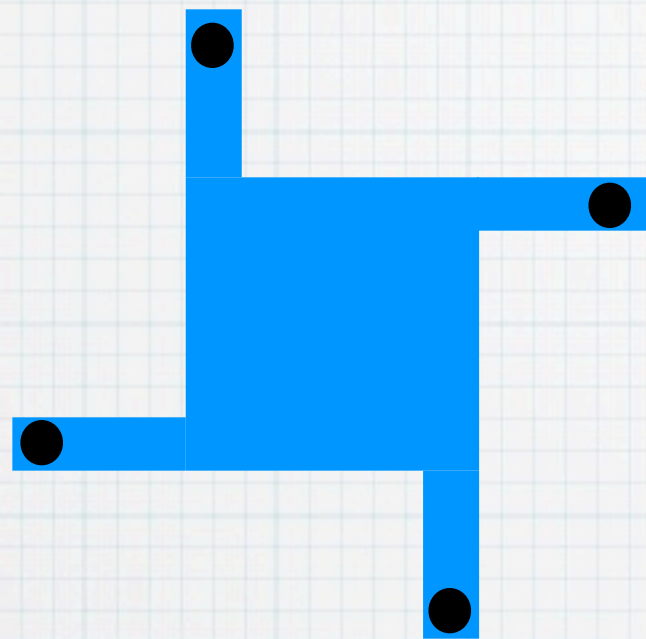
If the symmetry isn't perfect, and  $R_A$  is only approximately equal to  $R_B$ , then we can use the average value to get a reasonable estimate of the sheet resistance:

$$R_S \approx \frac{\pi}{\ln(2)} \left[ \frac{R_A + R_B}{2} \right]$$

If  $R_A$  and  $R_B$  are very different, then we must use the top relationship to find the sheet resistance. It's not trivial, but it's not hard either.



Usually, we try to make the sample symmetrical (a circle or a square), but we can never exactly meet the requirements that the contacts be negligibly small and be exactly at the periphery. To minimize these errors induced by the imperfect contacts, we can use slightly modified shapes.



The advantage to the vdP technique is that you don't have to do lithography to make the samples. It also lends itself well to Hall effect measurements, which allow you to determine carrier concentration and mobility independently. (Guest lecture, anyone?)

Question: Why not measure all four resistance configurations and take the average of those? Ans: Part of the vdP's proof was that the other two measurements are redundant, i.e.  $V_{34}/I_{12} = V_{12}/I_{34}$  and  $V_{23}/I_{14} = V_{41}/I_{23}$ .



# Example

In a van der Pauw measurement of a uniformly doped p-type layer in a silicon wafer, the two measured resistances are  $250\ \Omega$  and  $400\ \Omega$ . (Apparently, the sample was not very symmetric.) Determine the sheet resistance of the layer. If the layer is  $5\ \mu\text{m}$  thick, determine the wafer resistivity and doping concentration.

$R_A$  and  $R_B$  are not real close together, but we can try the approximate formula, using the average resistance value:

$$\begin{aligned} R_S &= \frac{\pi}{\ln(2)} \frac{R_A + R_B}{2} \\ &= \frac{\pi}{\ln(2)} \frac{250\Omega + 400\Omega}{2} = 1473 \frac{\Omega}{\square} \end{aligned}$$

Using the exact formula, which requires some numerical iteration, gives

$$R_S = 1445 \frac{\Omega}{\square} \quad \text{so the approximate formula was not so far off } (\approx 2\%).$$

The resistivity of the layer would be  $\rho = (1445\ \Omega)(0.0005\ \text{cm}) = 0.723\ \Omega\text{-cm}$ . From the graph (or the web calculator), the p-type doping concentration is  $N_A \approx 2.1 \times 10^{16}\ \text{cm}^{-3}$ .



# Example

Calculate the sheet resistance of an n-type layer formed by an n-type constant-source (Gaussian) diffusion with  $Q = 10^{14} \text{ cm}^{-2}$  and  $Dt = 10^{-9} \text{ cm}^2$ . The junction depth is  $2 \text{ }\mu\text{m}$ .

$$N(x) = \frac{Q}{\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right) = \left(1.78 \times 10^{18} \text{ cm}^{-3}\right) \exp\left(-\frac{x^2}{4 \times 10^{-9} \text{ cm}^2}\right)$$

$$\mu_n(x) = 92 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} + \left[1268 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}\right] \left[1 + \frac{N_D(x)}{1.3 \times 10^{17} \text{ cm}^{-3}}\right]^{-0.91}$$

$$\frac{1}{R_S} = q \int_0^{x_j} \mu_n(x) N_D(x) dx$$

There are no short cuts here. The only choice is to carry out a numerical integration. Your calculator might handle it. Matlab or Mathematica would will certainly do the calculation. I used a spreadsheet.

$$R_S = 224 \frac{\Omega}{\square}$$



# Example

Calculate the sheet resistance of an n-type layer formed by ion implantation. For the implant,  $Q = 10^{13} \text{ cm}^{-3}$ ,  $R_P = 0.2 \text{ } \mu\text{m}$ , and  $\Delta R_P = 0.05 \text{ } \mu\text{m}$ . The junction depth is  $0.4 \text{ } \mu\text{m}$ .

$$N(x) = \frac{Q}{\sqrt{2\pi} (\Delta R_P)} \exp \left[ -\frac{(x - R_P)^2}{2 (\Delta R_P)^2} \right] = \left( 8 \times 10^{17} \text{ cm}^{-3} \right) \exp \left[ -\frac{(x - 0.00002 \text{ cm})^2}{5 \times 10^{-11} \text{ cm}^2} \right]$$

$$\mu_n(x) = 92 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} + \left[ 1268 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right] \left[ 1 + \frac{N_D(x)}{1.3 \times 10^{17} \text{ cm}^{-3}} \right]^{-0.91}$$

Same deal again – use numerical integration.

$$R_S = 1504 \frac{\Omega}{\square}$$